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From black strings to black holes: nuttier and squashed AdS_5 solutions

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Abstract: We construct new solutions of the Einstein equations with negative cosmological constant in five spacetime dimensions. They smoothly emerge as deformations of the known AdS_5 black strings. The first type of configurations can be viewed as the $d = 4$ Taub-NUT-AdS solutions uplifted to five dimensions, in the presence of a negative cosmological constant. We argue that these solutions would provide the gravity dual for a $\mathcal{N} = 4$ super-Yang-Mills theory formulated in a $d = 4$ homogeneous Gödel-type spacetime background. A different deformation of the AdS_5 black strings leads to squashed AdS black holes and their topological generalizations. In this case, the conformal infinity is the product of time and a circle-fibration over a base space that is a two-dimensional Einstein space.

Keywords: Classical Theories of Gravity, Black Holes

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Contents

1 Introduction

Recently a tremendous amount of interest has been focused on solutions of the Einstein equations in more than $d = 4$ spacetime dimensions. This interest was enhanced by the development of string theory, which requires a ten-dimensional spacetime, to be consistent from a quantum point of view.

Solutions with a number of compact dimensions, present for $d \geq 5$ spacetime dimensions, are especially interesting, since they exhibit new features that have no analogue in the usual $d = 4$ theory. For the case $d = 5$ without a cosmological constant, the simplest configuration of this type is found by assuming translational symmetry along the extra coordinate direction and uplifting known solutions of the vacuum Einstein equations in four dimensions. This corresponds to a vacuum uniform black string (UBS) with horizon topology $S^2 \times S^1$. It approaches asymptotically the four dimensional Minkowski-space times a circle, the simplest case being the Schwarzschild black string.

Due to the absence of closed form solutions, relatively little is known about the generalizations of these configurations with a cosmological constant Λ. The $d = 5$ Anti-de Sitter (AdS) counterparts of the Schwarzschild black string have been considered for the first time in [1]. Their generalizations to higher dimensions $d > 5$ were discussed in [2], configurations with an event horizon topology $H^{d-3} \times S^1$ being considered as well. Various features of the AdS black strings[1], including solutions with matter fields, can be found in [4–6]. As argued in [1, 2], these solutions may have relevance in a AdS/CFT context, since they provide the gravity dual of a field theory on a $S^{d-3} \times S^1 \times R_t$ (or $H^{d-3} \times S^1 \times R_t$) background.

However, in four spacetime dimensions, the Schwarzschild black hole has an interesting generalisation, given by the famous Taub-NUT (TN) solution [7–9]. This (Lorentzian signature-) metric has become renowned for being '*a counterexample to almost anything*' [10], and played an important role in conceptual developments in general relativity [11]. The TN solution is usually interpreted as describing a gravitational dyon with both ordinary and magnetic mass. The nut charge n plays there a dual role to ordinary mass, in the same way that electric and magnetic charges are dual within Maxwell theory [12]. As discussed by many authors, the presence of magnetic-type mass introduces a "Dirac-Misner-string singularity" in the metric (but no curvature singularity). This can be removed by appropriate identifications and changes in the topology of the spacetime manifold, which imply a periodic time coordinate. The periodicity of the time coordinate prevents an interpretation of the TN metric as a usual black hole. Moreover, this metric is not asymptotically flat in the usual sense although it does obey the required fall-off conditions (see [13] for an extended discussion of the properties of the TN solution). The TN solution has also an interesting generalisation in heterotic string theory [14], with a full conformal field theory definition, which possibly indicates that string theory can very well live even in the presence of nonzero nut charge.

Of interest here are the generalisations of the four dimensional TN solution with a cosmological constant, which played an important role in (A)dS/CFT conceptual developments. For example, the vacuum TNAdS solution in four dimensions provided the first test bed for AdS/CFT correspondence in spacetimes where the asymptotic structure was only locally asymptotic AdS [15–17].

Similar to the Schwarzschild case, for $\Lambda = 0$, the four dimensional vacuum TN solution can trivially be uplifted to extend into the extra spacelike direction, which leads to a nuttier black object. However, this procedure cannot be repeated in the presence of a cosmological constant and the issue of constructing $d = 5$ counterparts of the known four dimensional TNAdS solutions has been scarcely explored in the literature. To our knowledge, the only known solutions possess the unusual feature that there is a constraint between the possible values of the nut charge and the cosmological constant [18] (see also their generalizations in [19]).

The main purpose of this work is to present a new family of solutions of $d = 5$ vacuum Einstein equations with a negative cosmological constant, which exist for any value of the cosmological constant. These solutions emerge smoothly from the known AdS_5 black strings and represent natural generalisations of the $d = 5$ AdS "nuttier" black holes in [18].

Since we could not find a general closed form solution, the configurations in this paper are constructed numerically by matching the near-horizon expansion of the metric to their

[1]These solutions should not be confused with the warped AdS configurations as discussed for instance in [3]. Although the warped solutions are also sometimes called black strings in the literature, their properties are very different as compared to those in [1, 2].

asymptotic Fefferman-Graham form [20]. Their global charges are computed by using the standard counterterm prescription. Our results indicate that the $TNAdS_5$ configurations share the basic properties of their $d = 4$ counterparts.

Moreover, these solutions may teach us something about the physics in spacetimes containing closed timelike curves (CTCs). Here we use the remark that the boundary metric of these $TNAdS_5$ solution is just the two-parameter family of $d = 4$ Gödel-type homogeneous spacetimes discussed in [21] (the famous acausal Gödel rotating universe [22] representing a particular case). Although the meaning of quantisation is unclear in the presence of CTCs, we argue that by using the AdS/CFT correspondence, one may get an idea about the behaviour of a quantum field theory in a such a background.

All configurations mentioned above have as basic building block the $d = 4$ TN solution with Lorentzian signature. However, the main relevance of the $d = 4$ nut-charged solutions is for a Euclidean signature of spacetime, in which case they have various applications in a quantum gravity context [23]. For $\Lambda = 0$, the $d = 5$ solution found by taking the product of a $d = 4$ Euclideanized TN metric with the time coordinate (a real line) represents the Gross-Perry-Sorkin monopole [24, 25], which plays an important role in the context of Kaluza-Klein theories. The Gross-Perry-Sorkin solution proves the existence there of asymptotically *locally* flat configurations, approaching a twisted S^1 bundle over a four dimensional Minkowski spacetime. Black hole solutions with this type of asymptotics enjoyed recently some interest, following the discovery by Ishihara and Matsuno (IM) [26] of a new charged solution in the five dimensional Einstein-Maxwell theory. The horizon of the IM black hole has S^3 topology, while its spacelike infinity is a squashed sphere or S^1 bundle over S^2. This solution has been generalised in various directions, including configurations with more general gauge fields [27] and stability analysis [28].

The AdS counterparts of the IM squashed black holes were considered in the recent paper [29]. These solutions have a number of interesting properties, providing the gravity dual for a $\mathcal{N} = 4$ super Yang-Mills theory on a background whose spatial part is a squashed three sphere.

The second purpose of this work is to present a detailed study of the AdS squashed black holes. We argue that they can also be viewed as smoothly emerging from the AdS_5 black strings by turning on a squashing parameter n, the usual Schwarzschild-AdS solution with a spherical horizon being found for a particular value of n. New $d = 5$ black hole solutions with a different topology of the event horizon are also presented. In particular, we discuss the asymptotic expansion of these configuration, compute their mass and discuss their thermodynamical features.

The outline of this article is as follows. In the next section we present the general framework while in section 3 we study the $TNAdS_5$ solutions. The AdS counterparts of the IM squashed black holes are discussed in section 4. We conclude in section 5 with some final remarks.

2 The general framework

2.1 The action and the counterterm method

We start with the following action in five spacetime dimensions

$$I_0 = \frac{1}{16\pi G}\int_{\mathcal{M}} d^5x\sqrt{-g}\,(R-2\Lambda) - \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^4x\sqrt{-\gamma}K, \tag{2.1}$$

where G is the gravitational constant, $\Lambda = -6/\ell^2$ is the cosmological constant, $\mathcal{M}$ is a five-dimensional manifold with metric $g_{\mu\nu}$, K is the trace of the extrinsic curvature $K_{ij} = -\gamma_i^k\nabla_k n_j$ of the boundary ∂M with unit normal n^j and induced metric γ_{ij}.

As usual with AdS solutions, it turns out that the action (2.1) evaluated at the level of the equations of motion diverges. The general remedy for this situation is to add counterterms, i.e. coordinate invariant functionals of the intrinsic boundary geometry that are specifically designed to cancel out the divergences[2]. Therefore, the following boundary counterterm part is added to the action (2.1) [30, 31]:

$$I_{\rm ct} = -\frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^4x\sqrt{-\gamma}\left\{-\left(\frac{3}{\ell}+\frac{\ell}{4}\mathcal{R}\right)+\log\left(\frac{r}{\ell}\right)\left(\frac{\ell^3}{8}\left(\frac{1}{3}\mathcal{R}^2-\mathcal{R}_{ij}\mathcal{R}^{ij}\right)\right)\right\}, \tag{2.2}$$

where $\mathcal{R}$, $\mathcal{R}_{ijkl}$ and $\mathcal{R}_{ij}$ are the curvature, Riemann and the Ricci tensor associated with the induced metric γ_{ij}. The second term in (2.2) is the usual expression required to cancel the logarithmic divergence that appears in $d = 5$ dimensions for some special boundary geometries, r being a coordinate normal to boundary.

Using these counterterms, one can construct a divergence-free boundary stress tensor from the total action $I = I_0 + I_{\rm ct}$ by defining a boundary stress-tensor:

$$\begin{aligned} T^{ij} = \frac{2}{\sqrt{-\gamma}}\frac{\delta I}{\delta\gamma_{ij}} = \frac{1}{8\pi G}\bigg\{ & K^{ij} - K\gamma^{ij} + \frac{\ell}{2}\mathcal{G}^{ij} + \log\left(\frac{r}{\ell}\right)\bigg[\frac{\ell^3}{8}\bigg(\frac{1}{3}\gamma^{ij}\mathcal{R}^2 - \gamma^{ij}\mathcal{R}_{kl}\mathcal{R}^{kl} \\ & -\frac{4}{3}\mathcal{R}\mathcal{R}^{ij} + 4\mathcal{R}^{ikjl}\mathcal{R}_{kl} + 2\Box\left(\mathcal{R}^{ij} - \frac{1}{2}\gamma^{ij}\mathcal{R}\right) + \frac{2}{3}\left(\gamma^{ij}\Box - \nabla^i\nabla^j\right)\mathcal{R}\bigg)\bigg]\bigg\}, \end{aligned} \tag{2.3}$$

(with $\mathcal{G}^{ij}$ the Einstein tensor of the boundary metric). Then a conserved charge associated with a Killing vector ξ^i at infinity can be calculated using the relationship

$$\mathfrak{Q}_\xi = \oint_\Sigma d^3S^i\xi^jT_{ij}, \tag{2.4}$$

where Σ is a closed surface. The conserved mass/energy M is the charge associated with the time translation symmetry, with $\xi = \partial/\partial t$.

Typically the boundary of this kind of spacetime will be an asymptotic surface at some large radius r. However the metric restricted to the boundary γ_{ij} diverges due to an infinite conformal factor r^2/ℓ^2, and so the metric upon which the dual field theory resides is defined using the rescaling

$$h_{ij} = \lim_{r\to\infty}\frac{\ell^2}{r^2}\gamma_{ij}. \tag{2.5}$$

[2] This technique is especially suited for the solutions discussed in this paper which present no obvious background.

JHEP08(2009)025

The stress-energy tensor $\langle\tau_{ij}\rangle$ for the dual theory formulated in a background metric given by h_{ij}, can be calculated by using the following relation [32]

$$\sqrt{-h}h^{ik}\langle\tau_{kj}\rangle = \lim_{r\to\infty}\sqrt{-\gamma}\gamma^{ik}T_{kj}. \tag{2.6}$$

2.2 Uniform black strings in AdS_5

Since all solutions discussed in this paper are connected with the UBS in [1, 2], we briefly present here their basic properties. A convenient metric ansatz to study this type of configurations is

$$ds^2 = \frac{dr^2}{f(r)} + r^2 d\Omega_k^2 + a(r)dz^2 - b(r)dt^2, \tag{2.7}$$

where $d\Omega_k^2 = d\theta^2 + F_k^2(\theta)d\varphi^2$ is the metric on a two-dimensional surface of constant curvature $2k$. The discrete parameter k takes the values $1, 0, -1$ and implies the form of the function $F_k(\theta)$:

$$F_k(\theta) = \begin{cases} \sin\theta, & \text{for} \quad k = 1 \\ \theta, & \text{for} \quad k = 0 \\ \sinh\theta, & \text{for} \quad k = -1. \end{cases} \tag{2.8}$$

Thus, for $k = 1$, θ and φ are the spherical coordinates with the usual range; for $k = 0, -1$ although φ is a periodic coordinate with $0 \le \varphi \le 2\pi$, the range of θ is not restricted. When $k = 0$, a constant (r, t) slice is a flat surface, while for $k = -1$, this sector is a space with constant negative curvature, also known as a hyperbolic plane. In what follows, V_k will denote the total area of the (θ, φ) surface. As usual with black strings, the coordinate along the compact direction is denoted by z and its asymptotic length is L (the value of L is arbitrary for the UBSs). The event horizon of a black string is located at $r = r_h$, where $f(r_h) = b(r_h) = 0$ and $a(r_h) > 0$. On a basic conceptual level, these solutions can be viewed as the four dimensional Schwarzschild-AdS_4 solutions 'uplifted' to $d = 5$ in the presence of a negative Λ. Different from the $\Lambda = 0$ case, the limit $r_h \to 0$ of the AdS black string solutions with an event horizon topology $S^{d-3} \times S^1$ corresponds to a nontrivial globally regular, soliton-like configuration.

The equations satisfied by the metric functions a, b, f are presented *e.g.* in [2]. Unfortunately no exact solution is known in the vacuum case[3] and thus one has to resort to numerical techniques. The expression of the solutions in the near horizon region and for large r can be found by taking $n = 0$ in the relations (3.3), (3.5) below.

Finally, by performing the double analytic continuation $t \to i\chi$, $z \to i\tau$, the black strings become static bubbles of nothing, with a line element

$$ds^2 = \frac{dr^2}{f(r)} + r^2(d\theta^2 + F_k^2(\theta)d\varphi^2) + b(r)d\chi^2 - a(r)d\tau^2. \tag{2.9}$$

The S^1 factor of the metric pinches off at the radius r_h, and the solution is regular if the spatial coordinate χ is identified with a period $\beta = 1/T_H$ (with T_H the Hawking temperature of the initial black string solution).

[3] Exact solutions have been found in Einstein-Maxwell-Λ theory [6, 33] for special values of the gauge coupling constant.

JHEP08(2009)025

2.3 The Taub-NUT-AdS$_4$ solution

For completeness, we briefly discuss here also the basic properties of the nut-charged Lorentzian AdS$_4$ solutions. For $\Lambda = 0$, the usual Taub-NUT construction corresponds to a U(1)-fibration over a two-dimensional Einstein space used as the base space. Usually taken to be a sphere, for $\Lambda = -3/\ell^2 < 0$ this space can also be a torus or an hyperboloid [16]. For a metric ansatz similar to that used in what follows for $d = 5$, the TNAdS$_4$ is given by[4]

$$ds^2 = \frac{dr^2}{f(r)} + r^2(d\theta^2 + F_k^2(\theta)d\varphi^2) - b(r)\left(dt + 4nF_k^2\left(\frac{\theta}{2}\right)d\varphi\right)^2, \tag{2.10}$$

where

$$\begin{aligned} b(r) &= k\left(1 - \frac{2n^2}{r^2}\right) + \frac{1}{r^2}\left(-2M\sqrt{r^2 - n^2} + \frac{1}{\ell^2}(r^4 + 4n^2r^2 - 8n^4)\right), \\ f(r) &= \left(1 - \frac{n^2}{r^2}\right) b(r), \end{aligned} \tag{2.11}$$

M being a parameter fixing the mass of solutions. For $k = 1$, in order to avoid a conical singularity at $\theta = \pi$, the coordinate t must be identified with period $8\pi n$ which yields a spacetime with CTCs. Although less obvious, all $k = 0$ solutions and the $k = -1$ metrics with $4n^2 > \ell^2$ are also not globally hyperbolic [34, 35].

It is also instructive to give the large r asymptotics of the metric functions in (2.10)

$$\begin{aligned} b(r) &= \frac{r^2}{\ell^2} + \left(k + \frac{4n^2}{\ell^2}\right) - \frac{2M}{r} + O(1/r^2), \\ f(r) &= \frac{r^2}{\ell^2} + \left(k + \frac{3n^2}{\ell^2}\right) - \frac{2M}{r} + O(1/r^2). \end{aligned} \tag{2.12}$$

In the limit of vanishing nut charge, the metric (2.10) describes topological black holes.

3 Generalized Taub-NUT-AdS$_5$ solutions

3.1 The ansatz and asymptotics

Unfortunately, there is no prescription to uplift a four dimensional solution to higher dimensions in the presence of a cosmological constant. However, the expressions (2.7), (2.10) above, naturally lead to the following metric ansatz for TNAdS$_5$ solutions:

$$ds^2 = \frac{dr^2}{f(r)} + r^2(d\theta^2 + F_k^2(\theta)d\varphi^2) + a(r)dz^2 - b(r)\left(dt + 4nF_k^2\left(\frac{\theta}{2}\right)d\varphi\right)^2, \tag{3.1}$$

with $k = \pm 1, 0$ and $F_k(\theta)$ still given by (2.8). The range of the coordinates θ, φ and z for this line element is similar to the black string case case.

[4] The usual form of the metric used in the literature is recovered by taking $r \to \sqrt{r^2 + n^2}$.

JHEP08(2009)025

The Einstein equations with a negative cosmological constant imply that the metric functions $a(r)$, $b(r)$ and $f(r)$ are solutions of the following equations:

$$\begin{aligned} f' &= \frac{2k}{r}+\frac{8r}{\ell^2}-\frac{2f}{r}-f\left(\frac{a'}{a}+\frac{b'}{b}\right)+\frac{4n^2b}{r^3}\,, \\ a'' &= \frac{2a}{r^2}-\frac{2ka}{r^2f}-\frac{4a}{\ell^2f}+\frac{2ab'}{rb}+\frac{a'}{r}-\frac{ka'}{rf}-\frac{4ra'}{\ell^2f}+\frac{a'b'}{2b}+\frac{a'^2}{a}-\frac{2n^2b(a+ra')}{r^4f}\,, \\ \frac{b'}{b} &= 2\frac{a\left[2\ell^2(k-f)+12r^2\right]-2r\ell^2fa'}{r\ell^2f\left[ra'+4a\right]}+\frac{4n^2ab}{r^3f(ra'+4a)}. \end{aligned} \tag{3.2}$$

We are interested in solutions with a nonextremal horizon located at $r=r_h$. As $r\to r_h$, the following approximate form of the metric functions holds:

$$\begin{aligned} a(r) &= a_h+a_1(r-r_h)+a_2(r-r_h)^2+O(r-r_h)^3, \\ b(r) &= b_1(r-r_h)+b_2(r-r_h)^2+O(r-r_h)^3, \\ f(r) &= f_1(r-r_h)+f_2(r-r_h)^2+O(r-r_h)^3, \end{aligned} \tag{3.3}$$

in terms of two positive parameters b_1, a_h. One finds *e.g.*

$$\begin{aligned} &f_1=\frac{k}{r_h}+\frac{4r_h}{\ell^2}, \qquad a_1=\frac{8a_hr_h}{4r_h^2+k\ell^2}, \qquad f_2=\frac{3b_1n^2}{2r_h^3}-\frac{k}{r_h^2}-\frac{2}{\ell^2}, \\ &a_2=\frac{4a_h(4r_h^3-b_1n^2\ell^2)}{r_h(4r_h^2+k\ell^2)^2}, \qquad b_2=-\frac{b_1(4r_h^3+b_1n^2\ell^2+2kr_h\ell^2)}{8r_h^4+2kr_h^2\ell^2}. \end{aligned} \tag{3.4}$$

The condition for a regular horizon is $f'(r_h)>0$, which for $k=-1$ implies the existence of a minimal value of r_h, i.e. $r_h>\ell/2$.

At large r, a straightforward computation show that the functions appearing in the metric admit the following expansion:

$$\begin{aligned} a(r) &= \frac{r^2}{\ell^2}+\frac{1}{2}\left(k+\frac{2n^2}{\ell^2}\right)+\frac{\ell^2}{r^2}\left(c_z+\frac{1}{12}\left(k+\frac{4n^2}{\ell^2}\right)\left(k+\frac{8n^2}{\ell^2}\right)\log\left(\frac{r}{\ell}\right)\right)+O\left(\frac{\log r}{r^4}\right), \\ b(r) &= \frac{r^2}{\ell^2}+\frac{1}{2}\left(k+\frac{4n^2}{\ell^2}\right)+\frac{\ell^2}{r^2}\left(c_t+\frac{1}{12}\left(k+\frac{4n^2}{\ell^2}\right)\left(k+\frac{16n^2}{\ell^2}\right)\log\left(\frac{r}{\ell}\right)\right)+O\left(\frac{\log r}{r^4}\right), \\ f(r) &= \frac{r^2}{\ell^2}+\frac{2}{3}\left(k+\frac{5n^2}{2\ell^2}\right)+\frac{\ell^2}{r^2}\left(c_t+c_z-\frac{n^2}{2\ell^2}\left(k+\frac{4n^2}{\ell^2}\right)+\frac{1}{6}\left(k+\frac{4n^2}{\ell^2}\right)\left(k+\frac{12n^2}{\ell^2}\right)\log\left(\frac{r}{\ell}\right)\right) \\ &\quad +O\left(\frac{\log r}{r^4}\right), \end{aligned} \tag{3.5}$$

which generalizes for $n\neq 0$ the expansion derived in [2, 6] (note the occurence here of log terms even for the planar case $k=0$).

Remarkably, the equations (3.2) admit an exact solution found by Mann and Stelea in [18] for $n^2=\ell^2/4$, $k=-1$ (this being the only case when the log terms are absent in the asymptotics (4.5)):

$$f(r)=\frac{r^2}{\ell^2}-\frac{1}{4}+\frac{\ell^2}{r^2}c_t-\frac{\ell^4}{4r^4}c_t, \quad a(r)=\frac{r^2}{\ell^2}-\frac{1}{4}, \quad b(r)=\frac{r^2}{\ell^2}+\frac{\ell^2}{r^2}c_t. \tag{3.6}$$

The horizon of this solution is at $r_h=\ell c_1$ (where $c_t=-c_1^4$) and has $a_h=c_1^2-1/4$ (this implies $c_1>1/2$).

JHEP08(2009)025

3.2 Global charges and general features

For n different from zero, the $k = 1$ metric structure (3.1) shares the same troubles exhibited by usual $d = 4$ TN metric, i.e. the solutions cannot be interpreted properly as black holes. Requiring the absence of singularities due to the Misner string implies a periodicity $8\pi n$ for the time coordinate. For $k = 0, -1$ the fibration is trivial and there is no Misner string singularity and no required periodicity for t. However, the pathology of CTCs still occurs for all $k = 0$ metrics and some of the $k = -1$ solutions. This can be seen *e.g.* by looking at the $g_{\varphi\varphi}$ component of the metric tensor

$$g_{\varphi\varphi} = r^2 F_k^2(\theta) - 16b(r)n^2 F_k^4(\theta/2) = 4F_k^2(\theta/2)\left(r^2(1-kF_k^2(\theta/2)) - 4n^2 b(r) F_k^2(\theta/2)\right). \quad (3.7)$$

For $k = 0$, $g_{\varphi\varphi}$ becomes negative for large enough values of θ. Thus the integral curves of the Killing vector $\partial/\partial\varphi$ provide an example of closed curves for a timelike motion. The case $k = -1$ is more involved, the sign of $g_{\varphi\varphi}$ being fixed by the ratio $4n^2 b(r)/r^2$. Although we could not prove that rigorously, our numerical results suggest that the value $4n^2/\ell^2$ (i.e. the Mann-Stelea solution (3.6)) always separates the $k = -1$ geometries with CTCs from those which admit a global time coordinate. (We recall that this is also the critical value for the $TNAdS_4$ metric (2.10), separating the $k = -1$ globally hyperbolic solutions from those presenting CTCs).

Although there is no clear definition of the global charges, Hawking temperature and entropy in the presence of CTCs, one can formally extend some results valid in the $k = -1$ globally hyperbolic case to the general situation (see *e.g.* the related discussion in [34, 36]).

Thus, as for UBSs, the nuttier solutions would possess two global charges–the mass M and tension $\mathcal{T}$, associated with the Killing vectors $\partial/\partial t$ and $\partial/\partial z$, respectively. Their values are fixed by the constants c_t and c_z, which enter the expression of the metric functions at infinity (3.5). To compute M and $\mathcal{T}$ we use the prescription (2.4), with the result

$$M = M_0 + M_c^{(k)}, \text{where } M_0 = \frac{\ell}{16\pi G}\left[c_z - 3c_t\right] L V_k, \text{and } M_c^{(k)} = \frac{V_k L \ell}{192\pi G}\left(k^2 + \frac{19n^2}{\ell^2}\left(\frac{7n^2}{\ell^2} + 2k\right)\right),$$

$$\mathcal{T} = \mathcal{T}_0 + \mathcal{T}_c^{(k)}, \quad \text{where} \quad \mathcal{T}_0 = \frac{\ell}{16\pi G}\left[3c_z - c_t\right] V_k, \quad \text{and} \quad \mathcal{T}_c^{(k)} = -\frac{V_k \ell}{192\pi G}\left(\frac{n^2}{\ell^2} + k\right)^2. \quad (3.8)$$

$M_c^{(k)}$ and $\mathcal{T}_c^{(k)}$ in the above relations are Casimir-like terms which appear also in the $k = \pm 1$ black string limit.

The Hawking temperature of these solutions, as computed from the surface gravity is $T_H = \sqrt{f'(r_h) b'(r_h)}/4\pi$. Of interest is also the horizon area, given by $A_H = r_h^2 \sqrt{a(r_h)} V_k L$. The entropy of the solutions can be identified with one quarter of A_H for globally hyperbolic solutions only (i.e. $k = -1, n^2 \leq \ell^2/4$). To our knowledge, the thermodynamics of a (Lorentzian signature–) acausal spacetime is an open problem.

3.3 Numerical results

In the absence of explicit solutions for the generic values of the parameters, we have solved the system of equations (3.2) numerically. The computation of mass and tension for these solutions is a nontrivial problem which requires a very good numerical accuracy, since the

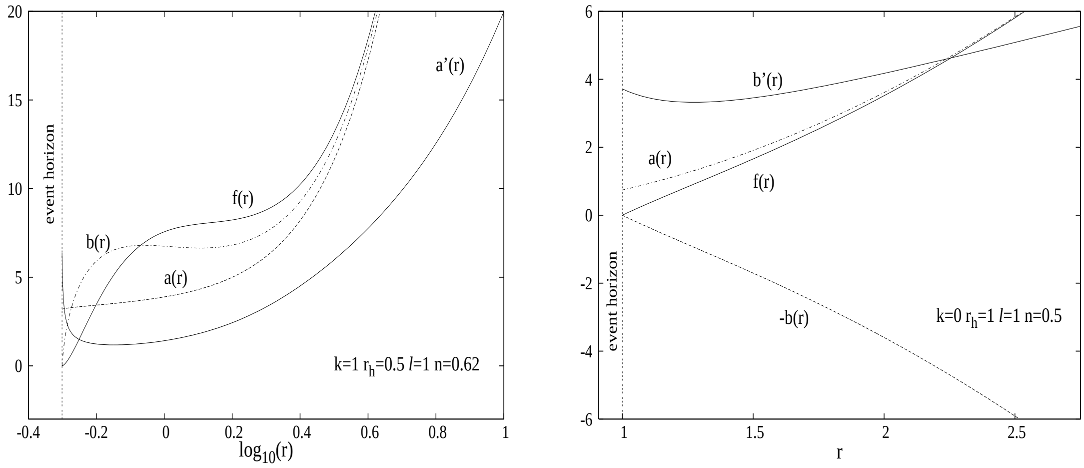

Figure 1. The profiles of typical $k = 1, 0$ nuttier solutions.

coefficients c_t, c_z appear as subleading terms in the asymptotic expansion (3.5) (see the ref. [6] for a detailed discussion of this point).

Following the methods in [2, 4], we have looked for solutions which have a regular horizon at $r = r_h$, treating the equations as a boundary value problem for $r \in [r_h, \infty]$ (thus we did not consider the behaviour of the solutions inside the horizon). The Einstein equations were solved by employing a collocation method for boundary-value ordinary differential equations, equipped with an adaptive mesh selection procedure [37]. Typical mesh sizes include $10^3 - 10^4$ points. The solutions have a typical relative accuracy of 10^{-8}.

Our numerical results clearly indicate the existence for any k of solutions of the equations (3.2), smoothly interpolating between the asymptotics (3.3) and (3.5). We have also verified that the solutions are singularity free for $r \geq r_h$. In particular, the Kretschmann scalar stays finite everywhere. In this approach, the input parameters are n, ℓ and r_h. The event horizon data $a(r_h)$, $b'(r_h)$ and the coefficients at infinity c_t and c_z are read from the numerical output.

In practice we fix the AdS length scale $\ell = 1$ and construct families of solutions by varying n and r_h for the three possible values of k. The choice $\ell = 1$ does not affect the generality of the results, since the cosmological constant can be arbitrarily rescaled by an appropriate redefinition of the radial variable r and of the parameter n. Also, since only n^2 appear in the equations, we shall restrict to positive values of n.

The solutions with $n = 0$ correspond to the UBSs discussed in [1, 2] and were used as initial guess in the numerical iteration. As typical examples of nut-charged solutions, we plot in figure 1 two solutions with $k = 1, 0$.

When increasing n, our numerical results show that the uniform black string gets continuously deformed. For a given value of the event horizon radius, the coefficients at infinity c_t, c_z as well as the horizon parameters T_H and $a(r_h)$ increase monotonically with n. The corresponding picture in this case is shown in figure 2 for both $k = 1$ and $k = -1$ solutions (the picture for $k = 0$ does not present new qualitative features). For the $k = 1$ case, when we continue to increase n, the numerical results clearly show that the derivative

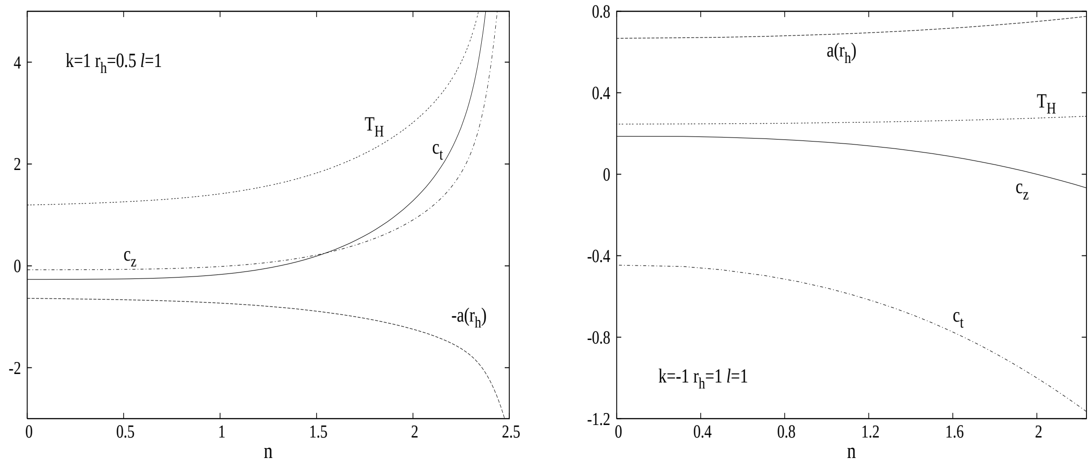

Figure 2. The dependence of the $k = 1, -1$ nuttier solutions on the parameter n for fixed event horizon radius r_h.

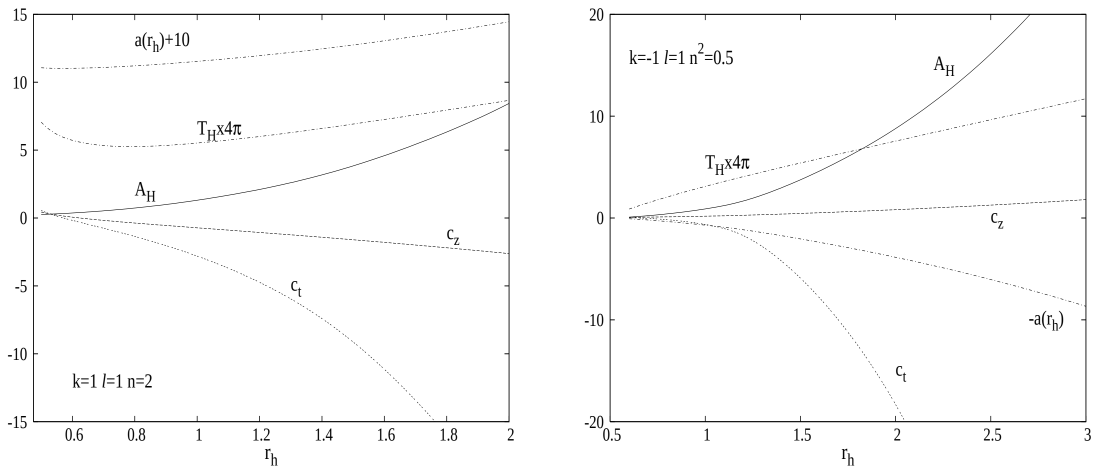

Figure 3. The dependence of the solutions on the event horizon radius r_h for $k = \pm 1$ nuttier solutions with fixed n .

of the function b at the horizon diverges (while the derivative of the function f remains constant) when a maximal value $n = n_c$ is approached. As a consequence, the surface gravity diverges in the limit $n \to n_c$. A similar maximal value of n is likely to exist also for $k = 0, -1$. However, in this case it has proven difficult to compute accurately the asymptotic coefficients c_t, c_z for large n.

As expected, for any value of $n \neq 0$, we notice also the presence of a minimal value $r_h^{(m)}$ of the event horizon radius. As $r_h \to r_h^{(m)}$ a critical solution is approached and the numerical solver fails to converge. The study of this critical solution seems to require a different parametrization of the metric ansatz and is beyond the purposes of this paper. Also, for any k and n, we did not find a maximal allowed value of the event horizon radius.

3.4 The boundary metric and the putative dual CFT

The line element of the boundary metric for the $d = 5$ TN solutions (3.1) is found by employing the prescription (2.5) and reads

$$ds^2 = \ell^2 \left(d\theta^2 + F_k^2(\theta)d\varphi^2\right) + dz^2 - \left(4nF_k^2\left(\frac{\theta}{2}\right)d\varphi + dt\right)^2. \tag{3.9}$$

The relation (2.6) predicts the following form of the stress tensor of the $\mathcal{N} = 4$ super-Yang-Mills theory formulated in this background, consisting in two parts:

$$\langle\tau_i^j\rangle = \langle\tau_i^{j(f)}\rangle + \langle\tau_i^{j(0)}\rangle, \tag{3.10}$$

with the nonzero components

$$\begin{aligned}
\langle\tau_z^{z(0)}\rangle &= -\frac{u}{24\ell}\left(k + \frac{n^2}{\ell^2}\right)^2, \\
\langle\tau_\theta^{\theta(0)}\rangle &= \langle\tau_\varphi^{\varphi(0)}\rangle = \frac{u}{12\ell}\left(\frac{n^2}{2\ell^2}\left(\frac{83n^2}{\ell^2} + 28k\right) + k^2\right), \\
\langle\tau_\varphi^{t(0)}\rangle &= -u\frac{n}{2\ell}F_k^2\left(\frac{\theta}{2}\right)\left(\frac{2n^2}{\ell^2}\left(\frac{36n^2}{\ell^2} + 11k\right) + k^2\right), \\
\langle\tau_t^{t(0)}\rangle &= -\frac{u}{24\ell}\left(\frac{19n^2}{\ell^2}\left(\frac{7n^2}{\ell^2} + 2k\right) + k^2\right),
\end{aligned} \tag{3.11}$$

and

$$\begin{aligned}
\langle\tau_z^{z(f)}\rangle &= \frac{u}{2\ell}(3c_z - c_t), & \langle\tau_\theta^{\theta(f)}\rangle &= \langle\tau_\varphi^{\varphi(f)}\rangle = -\frac{u}{2\ell}(c_t + c_z), \\
\langle\tau_\varphi^{t(f)}\rangle &= u\frac{8c_t n}{\ell}F_k^2\left(\frac{m\rho}{2}\right), & \langle\tau_t^{t(f)}\rangle &= \frac{u}{2\ell}(3c_t - c_z),
\end{aligned} \tag{3.12}$$

with $u = \frac{N^2}{4\pi^2\ell^3}$ (here we have replaced $8\pi G = 4\pi^2\ell^3/N^2$ [38], with N the rank of the gauge group of the dual $\mathcal{N} = 4,\ d = 4$ theory). This is the form of an anisotropic perfect fluid, the first part, $\langle\tau_i^{j(0)}\rangle$, encoding the Casimir-type background contribution. As expected, this stress tensor is not traceless,

$$\langle\tau_i^i\rangle = \frac{N^2}{4\pi^2\ell^3}\frac{(4n^2 + k\ell^2)^2}{12\ell^5}, \tag{3.13}$$

which matches exactly the conformal anomaly of the boundary CFT [31]:

$$\mathcal{A} = -\frac{2\ell^3}{16\pi G}\left(-\frac{1}{8}\mathcal{R}_{ab}\mathcal{R}^{ab} + \frac{1}{24}\mathcal{R}^2\right). \tag{3.14}$$

Here one should remark that, after defining a new "radial" coordinate $\rho = \ell\theta$ and the new parameters $m = 1/\ell,\ \Omega = n/\ell^2$, the background metric (3.9) upon which the dual field theory resides can be written as

$$ds^2 = d\rho^2 + \frac{F_k^2(m\rho)}{m^2}d\varphi^2 + dz^2 - \left(\frac{4\Omega}{m^2}F_k^2\left(\frac{m\rho}{2}\right)d\varphi + dt\right)^2. \tag{3.15}$$

This can be recognised as the standard form used in the literature of the homogeneous Gödel-type universe [21]. The famous Gödel rotating universe [22] corresponds to the case $k = -1,\ m^2 = 2\Omega^2$, (i.e. $\ell^2 = 2n^2$). The boundary spacetime of a $k = 0$ TNAdS_5 solution is known as the Som-Raychaudhuri spacetime [39] (the $d = 5$ counterpart of this metric is a supersymmetric solution [40], which enjoyed recently some interest in the literature). The value $m^2 = 4\Omega^2$ ($\ell^2 = 4n^2$) with $k = -1$ is special again, and we find the Rebouças-Tiomno space-time [41], which is the direct product of the AdS_3 spacetime and the $z-$direction, the spacetime symmetry being enhanced in this case.

The features of the $d = 4$ Gödel-type universes have been extensively discussed in the literature from various directions. Of interest here is the occurrence of CTCs for a range of the parameters (k, m, Ω) [21]. The situation here reflects that found for the $d = 5$ bulk spacetime. One can easily see that for $k = 1, 0$ the causality is violated for any (m, Ω). In the hyperbolic case $k = -1$, causality violation on the boundary metric (3.15) will appear only for $m^2 < 4\Omega^2$, the Rebouças-Tiomno space-time separating globally hyperbolic spacetimes from causality violating ones.

The violation of causality in a $k = 0, -1$ Gödel-type solution is made possible essentially because the metric coefficient $g_{\varphi\varphi}$ assumes negative values for $\rho > \rho_0$, (with $g_{\varphi\varphi}(\rho_0) = 0$), an effect induced by the nondiagonal metric term associated with rotation [11]. This is quite different from creating causal anomalies in flat space by identifying the time coordinate or from the standard AdS causal problems. A similar feature is shared by the Kerr black hole (for a region inside the horizon [11]) or by a Van Stockum infinitely long, rotating dust cylinder [42, 43]. However, different from the last cases, an acusal Gödel-type spacetime (3.15) is homogeneous [21], and there are CTCs through every event (hence the causality violation is not localized to some region). Also, a study of geodesics has shown that these spacetimes are geodesically complete (and thus singularity free) [44, 45].

The existence of CTCs renders the formulation of a physical theory in the backgrounds (3.15) rather obscure. The problems in the standard quantisation of a scalar field in a Gödel universe have been extensively discussed in [46]. The absence of a Cauchy surface results in the incompleteness of the mode solutions and thus in the impossibility to follow the standard quantisation procedure[5].

However, in principle one can use the AdS/CFT correspondence to predict qualitative features of a quantum field theory in a Gödel-type background with CTCs, despite the fact that the quantisation procedure is unclear in this case.

The issue is interesting especially in connection with the chronology protection conjecture [50]. This conjecture is usually enforced by the back reaction of the (divergent) energy momentum tensor of a test field on the spacetime geometry, via the semi-classical Einstein equations. Despite a number of attempts, the status of this conjecture in a Gödel-type background is still unclear.

[5]However, as argued in [35, 47], it may be possible to avoid some of these problems by using the Euclidean approach to the quantum field theory. A Euclidean section of the line element (3.15) is found by taking $\Omega \to ia$, $t \to i\tau$. For a general discussion of the Euclidean approach to field quantization for acausal spacetimes, see [48, 49].

JHEP08(2009)025

As one can see from (3.10), the stress tensor of the dual theory $\langle\tau_i^k\rangle$ is finite and well defined for any range of parameters[6]. In particular it stays finite for $\rho = \rho_0$ (where the Killing vector $\partial/\partial\varphi$ changes the sign for acausal $k = -1$ metrics). The finiteness of the stress tensor (3.10) implies that in this case the chronology protection conjecture cannot be settled at this level. This is likely to be connected with the fact that a non-globally hyperbolic Gödel-type spacetime is not the result of evolution of certain initial data, but rather it has existed "forever". Moreover, since there is no acausal geodesic motion in this background [44, 45], the standard arguments predicting a divergence of $\langle\tau_i^j\rangle$, based on a Hadamard form of the Green function do not apply.

A similar conclusion has been reached in [35] for the case of a four dimensional TNAdS metric (2.10). The boundary metric there is given by (3.15) with the dz^2 term suppressed. However, the structure of the boundary stress tensor is very different in that case from (3.10) (in particular $\langle\tau_i^i\rangle = 0$ for $d = 3$). Moreover, while the AdS_4/CFT_3 correspondence is still relatively poor understood, this is not the case for AdS_5/CFT_4.

For the solutions in this paper, a meaningful dual CFT should exist at least for the globally hyperbolic solutions with $k = -1$, $\ell^2 \geq 4n^2$. Concerning the other cases, one should mention that, following the discovery of $d = 5$ supersymmetric spacetimes with CTCs [40], there has been a lot of discussion in the literature whether spacetimes with CTCs are admissible backgrounds in string theory. Although it is tempting to claim that such solutions are simply not physical, to the best of our knowledge, this question has no clear answer to date. Most of the discussion has centered on some string versions of the chronology protection conjecture. Among them, Boyda et. al. [52] claimed that the chronology (and thus a quantum field theory) is well defined once the Gödel universe is prescribed with the macroscopic holography. It is also worthwhile to mention that the $d = 4$ Taub-NUT geometry can be embedded into heterotic string theory with a full CFT definition [53]. Thus it may be that string theory is able to make physical sense of the CTCs, despite their pathological character in general relativity [14].

On general grounds, one expects that a more detailed study of the $\mathcal{N} = 4$ super-Yang-Mills theory formulated in a $d = 4$ Gödel-type acausal background would reveal the existence of some pathological features. For example, it would be interesting to consider the effects of the higher order terms in the holographic stress tensor T_{ik} and to look for a hydrodynamical description of a gauge theory in the background (3.15). However, these aspects would require a study beyond the framework of this paper and are presently under study.

[6] See also the similar results in [51] for a class of supersymmetric solutions of $d = 5$ minimal gauged supergravity, containing CTCs beyond a critical value of an input parameter. In this case also, the holographic energy-momentum tensor remains finite even when the CTCs appear.

JHEP08(2009)025

4 Squashed AdS$_5$ black holes and their generalizations

4.1 The ansatz and asymptotics

A different class of solutions is found when deforming the AdS$_5$ black strings along the $z-$direction. The metric ansatz in this case is

$$ds^2 = \frac{dr^2}{f(r)} + r^2(d\theta^2 + F_k^2(\theta)d\varphi^2) + a(r)(dz + 4nF_k^2(\theta/2)d\varphi)^2 - b(r)dt^2. \qquad (4.1)$$

For $k = 1$, these are the solutions considered in [29] for a slightly different metric ansatz, representing the natural AdS counterparts of the $\Lambda = 0$ IM-type black holes [26]. Here we clarify their asymptotics (which was not considered in [29]), and discuss their thermodynamics.

It is obvious that no causal pathology appears here, t being a global time coordinate. For $k = 1$, the coordinate z becomes essentially an Euler angle, with a periodicity $L = 8\pi n$. Again, for $k = 0, -1$ the period of z is not fixed apriori. The range of θ, φ is similar to the black string case.

A suitable combination of the Einstein equations with a negative cosmological constant leads to the following equations for the metric functions $a(r)$, $b(r)$ and $f(r)$:

$$\begin{aligned}
f' &= \frac{2k}{r} + \frac{8r}{\ell^2} - \frac{2f}{r} - f\left(\frac{a'}{a} + \frac{b'}{b}\right) - \frac{4n^2a}{r^3},\\
b'' &= \frac{2b}{r^2} - \frac{2kb}{r^2f} - \frac{4b}{\ell^2 f} + \frac{2ba'}{ra} + \frac{b'}{r} - \frac{kb'}{rf} - \frac{4rb'}{\ell^2 f} + \frac{a'b'}{2a} + \frac{b'^2}{b} + \frac{2n^2a(b+rb')}{r^4 f}, \qquad (4.2)\\
\frac{a'}{a} &= 2\frac{b\left[2\ell^2(k-f) + 12r^2\right] - 2r\ell^2 fb'}{r\ell^2 f\left[rb' + 4b\right]} - \frac{4n^2ab}{r^3 f(rb' + 4b)}.
\end{aligned}$$

We are mainly interested in black-hole type solutions, with a horizon located at $r = r_h > 0$ (again, we shall not consider the behaviour of the solutions inside the horizon). Near a nonextremal horizon, the following power series expansion holds

$$\begin{aligned}
&f(r) = f_1(r-r_h) + f_2(r-r_h)^2 + O(r-r_h)^3, \quad b(r) = b_1(r-r_h) + b_2(r-r_h)^2 + O(r-r_h)^3, \qquad (4.3)\\
&a(r) = a_h + a_1(r-r_h) + a_2(r-r_h)^2 + O(r-r_h)^3.
\end{aligned}$$

Similar to the black string case, all coefficients here are fixed by $a(r_h)$ and $b'(r_h)$, the first terms being

$$\begin{aligned}
a_1 &= \frac{4a_h(2r_h^4 + a_hn^2\ell^2)}{4r_h^5 + r_h(kr_h^2 - 2a_hn^2)\ell^2}, \quad f_1 = \frac{4r_h}{\ell^2} + \frac{k}{r_h} - \frac{2a_hn^2}{r_h^3},\\
b_2 &= \frac{b_1(-8r_h^8 - 2r_h^4\ell^2(3kr_h^2 - 4a_hn^2) + (2a_h^2n^2 + 2ka_hn^2r_h^2 - k^2r_h^4)\ell^4)}{r_h(4r_h^4 - 2a_hn^2\ell^2 + kr_h^2\ell^2)^2}, \qquad (4.4)\\
f_2 &= \frac{-8r_h^8 + 2r_h^4\ell^2(-4a_hn^2 + 3kr_h^2) + (14a_h^2n^4 - 6ka_hn^2r_h^2 + k^2r_h^4)\ell^4}{4r_h^8\ell^2 + r_h^4\ell^4(kr_h^2 - 2a_hn^2)}.
\end{aligned}$$

The solutions have the following asymptotic form, which is found by solving the field equations for large r

$$
\begin{aligned}
a(r) &= \frac{r^2}{\ell^2}+\frac{1}{2}\left(k-\frac{4n^2}{\ell^2}\right)+\frac{\ell^2}{r^2}\left(c_z+\frac{1}{12}\left(k-\frac{4n^2}{\ell^2}\right)\left(k-\frac{16n^2}{\ell^2}\right)\log\left(\frac{r}{\ell}\right)\right)+O\left(\frac{\log r}{r^4}\right),\\
b(r) &= \frac{r^2}{\ell^2}+\frac{1}{2}\left(k-\frac{2n^2}{\ell^2}\right)+\frac{\ell^2}{r^2}\left(c_t+\frac{1}{12}\left(k-\frac{4n^2}{\ell^2}\right)\left(k-\frac{8n^2}{\ell^2}\right)\log\left(\frac{r}{\ell}\right)\right)+O\left(\frac{\log r}{r^4}\right),\\
f(r) &= \frac{r^2}{\ell^2}+\frac{2}{3}\left(k-\frac{5n^2}{2\ell^2}\right)+\frac{\ell^2}{r^2}\left(c_t+c_z+\frac{n^2}{2\ell^2}\left(k-\frac{4n^2}{\ell^2}\right)+\frac{1}{6}\left(k-\frac{4n^2}{\ell^2}\right)\left(k-\frac{12n^2}{\ell^2}\right)\log\left(\frac{r}{\ell}\right)\right)\\
&\quad +O\left(\frac{\log r}{r^4}\right),
\end{aligned}
\tag{4.5}
$$

depending again on two real parameters c_t, c_z.

The case $k = 1$, $4n^2 = \ell^2$ is special, the Schwarzschild-AdS$_5$ solution being recovered, with

$$
f(r) = b(r) = \frac{1}{4} + \frac{r^2}{\ell^2} + \frac{\ell^2}{r^2}c_t, \quad a = \frac{r^2}{\ell^2}\ . \tag{4.6}
$$

In this limit, the surface of constant r, t is a round sphere S^3. In the general $k = 1$ case, this surface is a squashed, topologically S^3 sphere. However, as $n \to 0$, the topology changes to $S^2 \times S^1$.

As discussed in [29], the $k = 1$ configurations possess a nontrivial globally regular limit as $r_h \to 0$. Making the assumption of regularity at $r = 0$ implies that this solution behaves near the origin as

$$
\begin{aligned}
a(r) &= \frac{1}{4n^2}r^2 + \frac{(1-f_2\ell^2)}{n^2\ell^2}r^4 + \ldots, \quad b(r) = b_0 + \frac{4b_0}{\ell^2}r^2 + \frac{16b_0(1-f_2\ell^2)}{3\ell^4}r^4 + \ldots,\\
f(r) &= \frac{1}{4} + f_2 r^2 + \left(\frac{11f_2}{3\ell^2} - \frac{2}{3\ell^4} - 3f_2^2\right)r^4 + \ldots,
\end{aligned}
$$

i.e. has two free parameters f_2, b_0. For $4n^2 = \ell^2$, the usual AdS$_5$ spacetime is recovered, with $f_2 = 1/\ell^2$, $b_0 = 1/4$ in this case.

By using the counterterm method in section 2, we find the following expressions for the mass of the solutions (which contains a Casimir term $M_c^{(k)}$):

$$
M = M_0 + M_c^{(k)}\,, \quad \text{where} \quad M_0 = \frac{\ell}{16\pi G}\left[c_z - 3c_t\right]LV_k, \quad M_c^{(k)} = \frac{V_k L\ell}{192\pi G}\left(\frac{n^2}{\ell^2} - k\right)^2. \tag{4.7}
$$

The Hawking temperature and the event horizon area of these solutions are given by

$$
T_H = \frac{1}{\beta} = \frac{\sqrt{f_1 b_1}}{4\pi}, \quad A_H = r_h^2 V_k L\sqrt{a_h}. \tag{4.8}
$$

Even in the absence of a closed form solution, the tree level Euclidean action I of these solutions can be evaluated by integrating the Killing identity $\nabla^\mu\nabla_\nu\zeta_\mu = R_{\nu\mu}\zeta^\mu$, for the Killing vector $\zeta^\mu = \delta_t^\mu$, together with the Einstein equation $R_t^t = (R - 2\Lambda)/2$. In this way, one isolates the bulk action contribution at infinity and at $r = r_h$ (or $r = 0$). The

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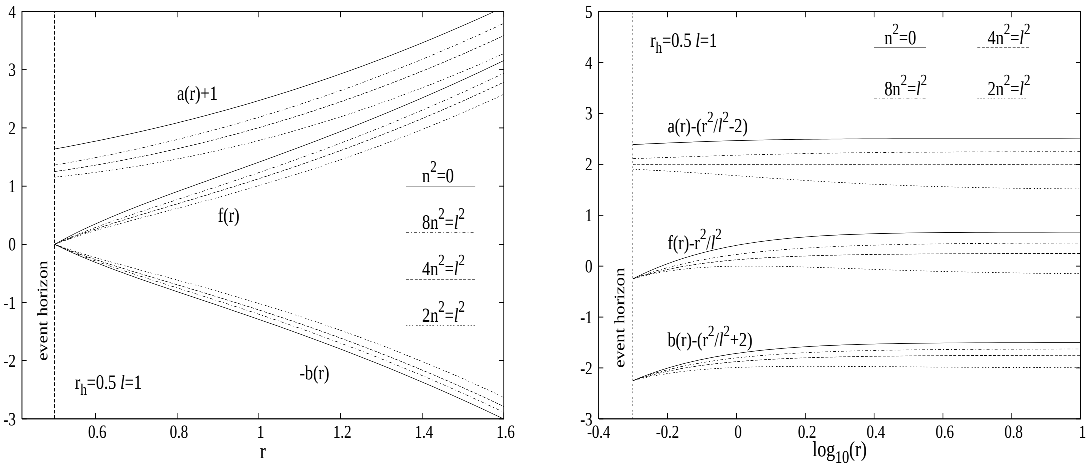

Figure 4. The profiles of typical $k = 1$ black hole solutions for several values of n.

divergent contributions given by the surface integral term at infinity are cancelled by the Gibbons-Hawking term in the action (2.1) together with the counterterms (2.2), and one finds a final expression for the total action in terms of boundary data at the horizon and at infinity. The entropy as computed from the Gibbs-Duhem relation $S = \beta M - I$ is $S = A_H/4G$, as expected.

4.2 Numerics and the properties of solutions

Again, in the absence of explicit solutions, we solved the system (4.2) numerically, as a boundary value problem. The methods empoyed here are similar to those described in section 3 and we shall not enter into details. Again, without any loss of generality, the AdS length scale ℓ is set to one and we consider only positive values of n.

As in the case of nuttier configurations, our method confirms the existence, for any k, of families of solutions labeled by r_h and n. In the limit $n \to 0$, the uniform black strings are recovered. Again, the AdS length scale ℓ is set to one, without any loss of generality.

Given its potential relevance in AdS/CFT, the case of main interest here is $k = 1$. Confirming the results in [29], we could construct a family of solutions interpolating between the AdS_5 uniform black strings and the Schwarzschild-AdS_5 black holes. Solutions with $n > \ell/2$ were considered as well. Profiles corresponding to $r_h = 0.5$ and several values of n are presented in figure 4 for the region close to horizon (left) and for large values of r (right). In figure 5 (left) we show the solutions' dependence on n for $r_h = 0.5$ (a similar picture was found for several other values of the event horizon radius).

Concerning the thermal properties, our numerical results indicate that the physics familiar from the Schwarzschild-AdS_5 case is valid also for these $k = 1$ squashed black holes. Their temperature is bounded from below for any n, and we have two branches consisting of smaller (unstable) and large (stable) black holes (see figure 6 (left)). Thus, at low temperatures we have a single bulk solution, which corresponds to the thermal globally regular solution (whose temperature may take arbitrary values). The free energy $F = I/\beta$ of the $k = 1$ solutions is positive for small r_h and negative for large r_h. This shows that the phase transi-

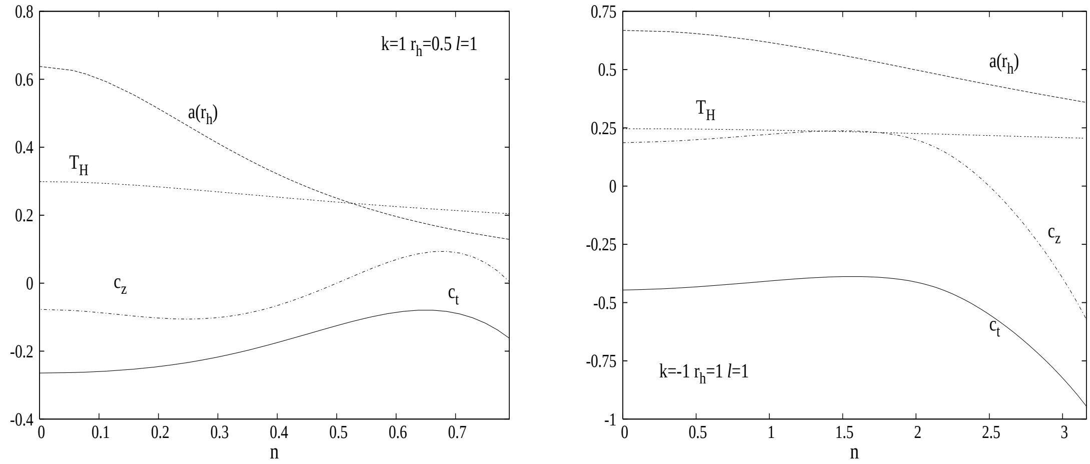

Figure 5. The dependence of the $k = 1, -1$ black hole solutions on the parameter n for fixed event horizon radius r_h.

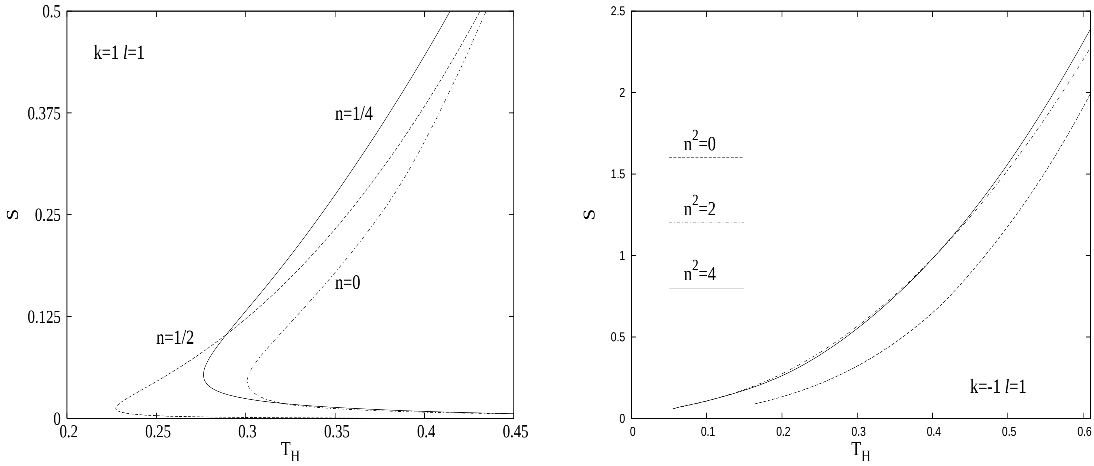

Figure 6. The entropy is plotted as a function of the Hawking temperature for $k = 1, -1$ black holes.

tion found in [54] occurs also in this case, as already conjectured in [29]. This is illustrated in figure 7, where the free energy is plotted versus the temperature for several values of n.

We have also considered solutions with $k = -1, 0$, corresponding to topological black holes. These types of configurations have a number of common features with their $k = 1$ counterparts. For example, the typical shape of the metric functions is similar to those presented in figure 4. However, for any value of n, the $k = -1, 0$ solutions exist only for large enough values of the horizon radius r_h. Their thermal properties are also different, as shown for the $k = -1$ solutions in figure 6 (right). In this case we noticed the existence of only one branch of thermally stable black holes, i.e. their entropy increases with the temperature. This is the behaviour found in [2] for the $n = 0$ topological UBS limit (this feature is also familiar from the study of the usual Schwarzschild-AdS topological black holes).

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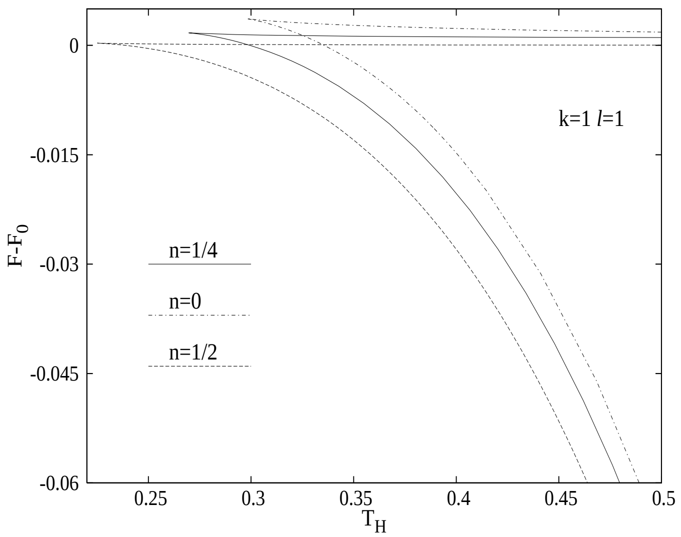

Figure 7. The free energy vs. the temperature for the small and large $k = 1$ squashed black hole solutions is plotted for three values of n. Here we have subtracted the free energy contribution F_0 of the corresponding globally regular solutions.

The dependence on the parameter n of a number of global quantities is plotted in figure 5 (right) for $k = -1$ solutions with $r_h = 1$.

4.3 The boundary metric and the dual CFT

In the asymptotic region, the $k = 1$ metric (4.1) becomes

$$ds^2 = \frac{\ell^2}{r^2}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2 + \frac{4n^2}{\ell^2}(d\psi + \cos\theta d\varphi)^2 - \frac{1}{\ell^2}dt^2\right), \tag{4.9}$$

where $\psi = \varphi - z/(2n)$. One can recognize that a surface of constant (r,t) represents a squashed three sphere, with $4n^2/\ell^2$ parametrising the squashing. The value $n = 2\ell$ separates prolate metrics from the oblate case $(n > 2\ell)$. Remarkably, this asymptotic metric is still maximally symmetric, i.e. to leading order $R_{ijkl} = -1/\ell^2(g_{ik}g_{jl} - g_{il}g_{jk})$ (this holds also for $k = 0, -1$ metrics in the large r limit).

The general form of the four dimensional boundary metric is given by the line element

$$ds^2 = \ell^2(d\theta^2 + F_k^2(\theta)d\varphi^2) + (dz + 4nF_k^2(\theta/2)d\varphi)^2 - dt^2, \tag{4.10}$$

which is a homogeneous spacetime with five Killing vectors, the $k = 1$ case corresponding to the frozen mixmaster universe.

Some features of the boundary CFT for $k = 1$ squashed black hole metrics were already discussed in [29]. Here we give the expression of the boundary stress tensor for dual CFT in a background metric (4.10):

$$\langle\tau_a^b\rangle = \langle\tau_a^{b(f)}\rangle + \langle\tau_a^{b(0)}\rangle, \tag{4.11}$$

JHEP08(2009)025

with the nonvanishing components[7]

$$
\begin{aligned}
\langle\tau_z^{z(0)}\rangle &= -\frac{u}{24\ell}\left(\frac{19n^2}{\ell^2}\left(\frac{7n^2}{\ell^2}-2k\right)+k^2\right),\\
\langle\tau_\theta^{\theta(0)}\rangle &= \langle\tau_\varphi^{\varphi(0)}\rangle = \frac{u}{12\ell}\left(\frac{n^2}{2\ell^2}\left(\frac{83n^2}{\ell^2}-28k\right)+k^2\right),\\
\langle\tau_\varphi^{z(0)}\rangle &= -u\frac{n}{2\ell}\left(\frac{18n^2}{\ell^2}-k\right)\left(\frac{4n^2}{\ell^2}-k\right)F_k^2(\theta/2),\\
\langle\tau_t^{t(0)}\rangle &= -\frac{u}{24\ell}\left(\frac{n^2}{\ell^2}-k\right)^2,
\end{aligned}
\tag{4.12}
$$

and

$$
\begin{aligned}
\langle\tau_z^{z(f)}\rangle &= u\frac{3c_z-c_t}{2\ell}, &\qquad \langle\tau_\theta^{\theta(f)}\rangle &= \langle\tau_\varphi^{\varphi(f)}\rangle = -u\frac{c_t+c_z}{2\ell},\\
\langle\tau_\varphi^{z(f)}\rangle &= u\frac{8c_z n}{\ell}F_k^2\left(\frac{\theta}{2}\right), &\qquad \langle\tau_t^{t(f)}\rangle &= u\frac{3c_t-c_z}{2\ell},
\end{aligned}
\tag{4.13}
$$

where, again, $u = N^2/4\pi^2\ell^3$. One can easily verify that the trace of this tensor matches exactly the conformal anomaly of the boundary CFT.

As discussed in [29], the thermodynamical entropy of the $\mathcal{N}=4$ super-Yang-Mills theory formulated in a $k=1$ background (4.10) agrees with that of the $k=1$ squashed black holes up to a factor of 3/4. It would be interesting to extend the results in [29] to the cases $k=0,-1$. This computation looks possible, since the line-element (4.10) is homogeneous for any k, which allows an explicit computation of the spectrum for scalar, gauge or spinor fields. A computation of the $\langle\tau_i^k\rangle$ tensor for the $\mathcal{N}=4$ super-Yang-Mills theory is another reasonable task, at least for $k=1$, given the existence in the literature of a number of partial results in this case [55].

5 Further remarks

In this paper we have presented arguments for the existence of two different generalizations of the known AdS_5 black string solutions. The first type of configuration, discussed in section 3, can be interpreted as the uplifted version of four dimensional TNAdS solutions and inherits most of the properties in that case. In particular, only a restricted set of solutions with a hyperbolic base space are free of causal pathologies.

An interesting question here is the possibility of the existence of more general nuttier solutions with a dependence on the coordinate z. These configurations would be intrinsic five-dimensional, and would present new qualitative features. Indeed, such configurations are known to exist for $k=1$ in the $n=0$ limit, and describe static nonuniform AdS black strings [56, 57]. This issue is related to the classical stability of the uniform solutions against

[7]Not by accident, this form reminds of the stress tensor (3.10)–(3.12) in a Gödel-type background. The reason is that line-elements (3.9), (4.10) present the same Euclidean section and are related through a simple analytic continuation involving also the parameter n. Thus the expressions of $\langle\tau_a^b\rangle$ are also related. A similar situation appears *e.g.* for the case of Misner spacetime and the infinitely long cosmic string solution [49].

small perturbations. The results in [58] indicate that the small AdS black strings whose conformal infinity is the product of time and $S^{d-3}\times S^1$ possess a classical Gregory-Laflamme instability [59]. Their nuttier generalizations are likely to exhibit this instability as well, at least for some range of the parameters. This would imply also the existence of a complicated phase structure of the solutions for the same set of boundary conditions, with a new branch of $d = 5$ nuttier nonuniform solutions, possessing a bulk dependence on both r and z.

In section 4, we discussed the basic properties of another type of solutions representing $d = 5$ squashed black holes and their topological generalizations. The main result there is a proof that the phase transition found in [54] for AdS_5 black holes with spherical horizon occurs also for solutions with a squashed horizon.

Our solutions in sections 3 and 4 can be used as new test grounds for the AdS/CFT correspondence. They have a nontrivial boundary structure which is generically a circle fibration over a base space that can have exotic topologies. Moreover, by using the relations in [60], they can be uplifted to $d = 10$ to become solutions of type IIB supergravity. Therefore their study could shed some light on the study of CFTs on some unusual backgrounds. In particular, one could be able to understand the thermodynamic phase structure of CFTs by working out the corresponding phase structure of the solutions in the bulk. An unexpected result here is the emergence of the $d = 4$ Gödel spacetime as the boundary metric of a particular nuttier solution. This may lead to some progress in the issue of field quantization in the presence of causal pathologies.

We close this work with several remarks on possible extensions of the solutions in this paper. For example, in section 3 we have considered the Lorentzian form of the nuttier configurations. However, the main relevance of the NUT charged solutions is for a Euclidean signature of spacetime, which would allow a study of their thermodynamics. This may be an interesting subject, since the results in [35] suggest that the entropy/area relation is always violated in the presence of a NUT charge. In principle, the Euclidean section of the nuttier solutions is simply obtained using the analytic continuations $t \to i\chi$ and $n \to in$. However, in the absence of a general exact solution, this is not useful in practice[8], and one has to employ again numerical methods to study the Euclidean counterparts of the solutions in section 3. One should also remark that, on the Lorentzian section, the mass and NUT parameter are unrelated. That is, once we fix the event horizon radius, the parameter n can be freely specified. However, the $k = 1$ Euclidean solutions have to satisfy one extra regularity condition, since no conical singularity should appear both at $r = r_h$ and $\theta = \pi$. This implies the relation $8\pi pn = 4\pi/\sqrt{f'(r_h)b'(r_h)}$, with p an integer [18]. From a numerical point of view, this would restrict the allowed values of r_h for a given n, and thus the possible values of M. Moreover, the possible existence of both "nut" and "bolt" configurations for a Euclidean signature is likely to lead to a complicated landscape of solutions there.

The analytic continuation $z \to i\tau$, $t \to i\chi$ and $n \to in$ of a nut-charged solution in

[8]This can easily be seen looking *e.g.* at the $d = 4$ closed form solution (2.10), (2.11). The metric functions $f(r)$, $b(r)$ look different on the Lorentzian and Euclidean sections.

section 3 leads to another interesting type of configuration, with a line element

$$ds^2 = \frac{dr^2}{f(r)} + r^2(d\theta^2 + F_k^2(\theta)d\varphi^2) + b(r)\left(4nF_k^2\left(\frac{\theta}{2}\right)d\varphi + d\chi\right)^2 - a(r)d\tau^2, \qquad (5.1)$$

(we recall that the functions $f(r), b(r)$ vanish for some $r = r_h > 0$, while $a(r_h) > 0$; thus these configurations are different from the squashed black holes in section 4). These solutions can be interpreted as nuttier deformation of the AdS bubbles (2.9). For $k = 1$ and $n = \ell/2$, an exact solution here reads

$$f(r) = \frac{r^2}{\ell^2} + \frac{1}{4} + \frac{\ell^2}{r^2}c_t + \frac{\ell^4}{4r^4}c_t, \quad b(r) = \frac{r^2}{\ell^2} + \frac{\ell^2}{r^2}c_t, \quad a(r) = \frac{r^2}{\ell^2} + \frac{1}{4}, \qquad (5.2)$$

which is the AdS_5 soliton discussed in [61]. The above configuration if free of singularities (apart from the central one at $r = 0$) for $c_t = -(4p^2 - 1)^2/(256p^4)$, with p an arbitrary integer. As argued in [61], this solution is the lowest energy state within that asymptotic class. The results in this work suggest the existence of similar AdS_5 solitons for any value of the ratio n/ℓ. We hope to return elsewhere to this subject.

Moreover, one expects other types of nut charged solutions to exist. For example, in principle one can use the globally hyperbolic solutions in section 4 to generate new nuttier solutions by using the analytic continuation $t \to it$, $n \to in$, $z \to iz$ for the line element (4.1). The asymptotic structure of the resulting configurations would be similar to that of the solutions in section 3. However, they would satisfy a different set of boundary conditions than (3.3) at $r = r_h$, with $a(r_h) = 0$ and $b(r_h) > 0$. Again, any progress in this direction appears to require a separate numerical study of the solutions.

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JHEP08(2009)025